P008 True amplitude Gaussian beam imaging

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Abstract

We present an original approach of true amplitude seismic imaging by means of weighted summation of multi-shot multi-offset data. These weights are computed by Gaussian beam (GB) tracing from current points within target area towards acquisition system. The special choice of GB provides possibility to take into account both illumination conditions and structure of covering layers. Global GB regularity allows person to be not care of caustics and foci locations and to use uniform relations through the target area and overburden layers.

Introduction.

Our aim is to image scatterers/reflectors in the Earth for a given macrovelocity model. There are plentiful theories and numerical algorithms designated to achieve this goal. Following [5], let us split all these approaches on two families – kinematic imaging and dynamic imaging and let us deal with the second one. Next, within this family we are interesting in amplitude preserving imaging procedures, that is with procedures providing images being free from influence of illumination conditions and geometrical spreading factor.

The common approaches to construct the desired image can be traced to the papers [2,3] and are based on a different kind of asymptotic expansion of Green's function in heterogeneous media. Up to the recent time the mainly used approximation is zero order ray expansion being valid for regular rays configuration only. The migration procedure being free from this shortcoming is Hill's Gaussian Beams (GB) migration [4]. It is based on representation of Green's function as a GB superposition being valid both for regular and irregular rays configurations. But at the moment there are a few attempts to develop preserving amplitude version of GB migration [1].

The approach presented below is quite different from all others because it does not use for true-amplitude imaging any asymptotic expansion of Green's function at all. Our version of stacking integral involves single GB, not their superposition. True/preserving amplitude imaging happens to be the intrinsic property of the proposed imaging procedure that is not needed in any additional computations and storage of true amplitude kernels and Hessians.

Statement of the problem

The wave propagation velocity below is supposed to be decomposed as macro-velocity $c_0(x,z)$ and reflectivity/scatterer component $c_1(x,z)$. In order to describe scattered/reflected wave field $u_1(x,z;x_s,0;\omega)$ Born's approximation is used. Input multi-shots multi-offsets data will be dealt with later are multi-shot multi-offset reflected/scattered waves $\varphi(x,x_s,\omega) = u_1(x,0;x_s,0;\omega)$.

True/preserving amplitudes imaging in the consequent will be treated as a procedure providing images with intensities being proportional to sharpness of the background perturbations $\dfrac{c_1(x,z)}{c_0(x,z)}$.

Description of the method

Let us choose some current interior point $(\bar{x},\bar{z})$ within target area and shoot from this point couple of rays toward free surface for given macro velocity model (see Fig.1) and construct a couple of GB - $u^{(gb)}(x,z;\bar{x},\bar{z};x_{0g,s};\omega)$ - attached to these rays as described in ([7], chapter 8). Here $(x_{0g,s},0)$ are points where these rays meet free surface. Now, taking into account that GB is ***a solution*** to the Helmholtz's equation and applying Green's theorem twice one comes to the following integral identity:

$$\int_{z=0} \frac{\partial u^{(gb)}(x_s,z;\bar{x},\bar{z};x_{0s};\omega)}{\partial z}\Big|_{z=0} dx_s \int_{z=0} \frac{\partial u^{(gb)}(x_s,z;\bar{x},\bar{z};x_{0g};\omega)}{\partial z}\Big|_{z=0} \varphi(x,x_s,\omega)dx =$$
$$= 2\omega^2 F(\omega)\iint_{z>0} \frac{1}{c_0^2(\xi,\eta)}\frac{c_1(\xi,\eta)}{c_0(\xi,\eta)} u^{(gb)}(\xi,\eta;\bar{x},\bar{z},x_{0s};\omega)u^{(gb)}(\xi,\eta;\bar{x},\bar{z},x_{0g};\omega)d\xi d\eta \qquad (1)$$

The product of GB in the right hand side of this identity is vanishing outside of intersection of hyperbolic vicinities of introduced above rays (fig.1). The diameter of this vicinity is about dominant wavelength. So, in the right hand side of (1) one can perform integration over this vicinity only and use within this vicinity constant macro velocity model $c_0(\xi,\eta)\cong c_0(\bar{x},\bar{z})$. This provides one with advantage to use explicit formula for GB within homogeneous media (see [7], chapter 8) rewritten in polar coordinates originated at current point $(\bar{x},\bar{z})$ as:

$$u_{0g}^{(gb)}(\xi,\eta;\bar{x},\bar{z};x_{0g(s)};\omega) = \exp\left\{ i\omega\left[\tau_{0g}(\bar{x},\bar{z}) - \frac{\rho\cos(\varphi-\varphi_{0g(s)})}{c_0(\bar{x},\bar{z})}\right]\right\}\times$$
$$\times\exp\left\{ -\frac{i\omega}{2c_0(\bar{x},\bar{z})}\frac{\rho^2\sin^2(\varphi-\varphi_{0g(s)})}{\rho\cos(\varphi-\varphi_{0g(s)})+\frac{ik}{2}\frac{c_0(\bar{x},\bar{z})}{\omega}}\right\}$$

It should be underlined, that this possibility is provided by special choice of mentioned above Gaussian beams – they possess their narrowest part at the current point $(\bar{x},\bar{z})$. Now in order to get the true/preserving amplitude image at the current point one should multiply both sides of (1) by $\exp\{-i\omega(\tau_{0g}(\bar{x},\bar{z})+\tau_{0s}(\bar{x},\bar{z}))\}$ and integrate with respect to ω:

$$\Phi(\bar{x},\bar{z};\varphi_{0g},\varphi_{0s}) \overset{def}{=} \left|\int_{\omega} \exp\{-i\omega(\tau_{0g}(\bar{x},\bar{z})+\tau_{0s}(\bar{x},\bar{z}))\}d\omega \int_{z=0}\frac{\partial u_{0s}^{(gb)}(x_s,z;\bar{x},\bar{z};x_{0s};\omega)}{\partial z}\Big|_{z=0}dx_s\right.$$
$$\left.\int_{z=0}\frac{\partial u_{0g}^{(gb)}(x_s,z;\bar{x},\bar{z};x_{0g};\omega)}{\partial z}\Big|_{z=0}\varphi(x,x_s,\omega)dx_s\right| =$$
$$= \left|\frac{2}{c_0^2(\bar{x},\bar{z})}\int_0^R d\rho\int_0^{2\pi} K(\rho,\varphi;\varphi_{0g},\varphi_{0s})\frac{c_1(\bar{x}+\rho\cos\varphi,\bar{z}+\rho\sin\varphi)}{c_0(\bar{x},\bar{z})}d\varphi\right|$$

As one can see, this is nothing else, but convolution of the sharpness of background perturbation with "smoothing" kernel:

$$K(\rho,\varphi;\varphi_{0g},\varphi_{0s}) = \int_{\omega} F(\omega)\exp\left\{-i\omega\rho\frac{\cos(\varphi-\varphi_{0g})+\cos(\varphi-\varphi_{0s})}{c_0(\bar{x},\bar{z})}\right\}\times$$
$$\times\exp\left\{-\frac{i\omega\rho^2}{2c_0(\bar{x},\bar{z})}\left(\frac{\sin^2(\varphi-\varphi_{0g})}{\rho\cos(\varphi-\varphi_{0g})+\frac{ik}{2}\frac{c_0(\bar{x},\bar{z})}{\omega}}+\frac{\sin^2(\varphi-\varphi_{0s})}{\rho\cos(\varphi-\varphi_{0s})+\frac{ik}{2}\frac{c_0(\bar{x},\bar{z})}{\omega}}\right)\right\}d\omega \qquad (2)$$

The smoothing kernel is determined by the ***local*** property of macro velocity model and does not depend on the illumination conditions and structure of the upper layers. It happens because the special choice of GB with shooting from the bottom is used as weights in (1) – they are computed for the given macro velocity model and take into account its global variability already.

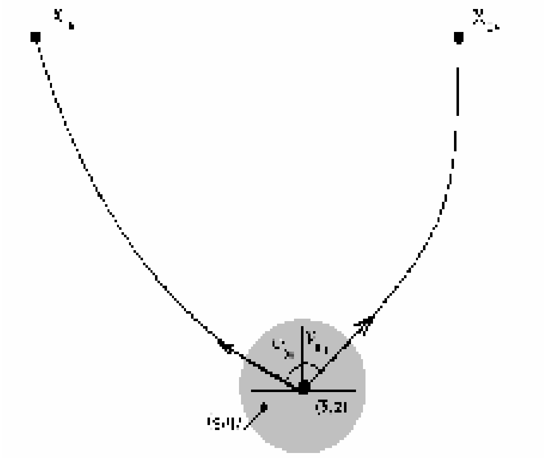

Fig.1. Geometry of true/preserving amplitude GB imaging

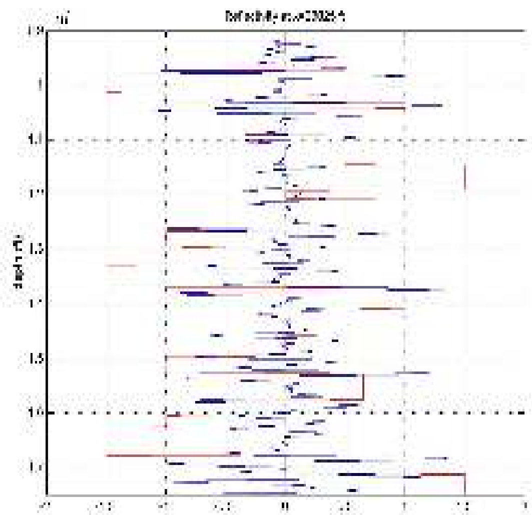

Fig.2. Recovered (blue) and true (red) reflectivity

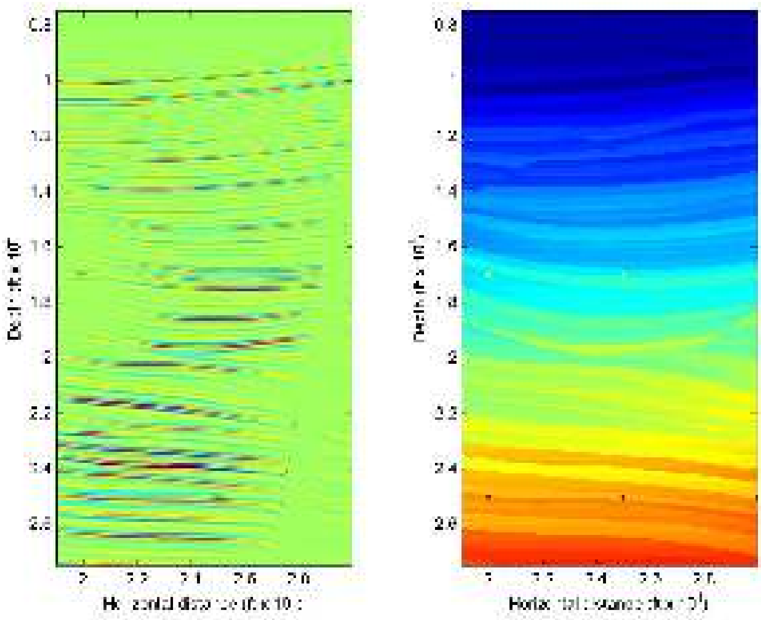

Fig.3. Recovered (left) and real (right) easy pieces of Sigsbee2A model.

Synthetic data processing

In order to illustrate presented above approach of true/preserving amplitude imaging synthetic SIGSBEE2A data set generated by SMAART Joint Venture were processed. On the Fig.2 one can see recovered (blue) and true (red) reflectivity taken along vertical line at x=26025 ft. There is perfect coincidence of jumps positions and amplitudes for both these reflectivities. Next, on the Fig.3 there are presented both recovered and real structures of the easy piece of Sigsbee2A model. As easy we mean the piece free from the salt body. The strip on the bottom right of the recovered part is produced by the GB diffraction on the left salt flank. The quality of subsalt recovery one can see on the Fig.4. There is again perfect matching of recovered and real structures. Another important property of the proposed approach is its ability to stress reflected/scattered elements with specific orientations. The more careful analysis of smoothing kernel (2) proves that the brightest on the image are elements possessing normal being bisectrix of the angle formed by the chosen couple of rays. On the Fig.5 one can see image formed by the rays oriented to imaging of steep reflecting elements. As one can see there is almost no presence of regular interfaces but fault only. The image of point sactterers is still present because of uniform character of their dispersion index.

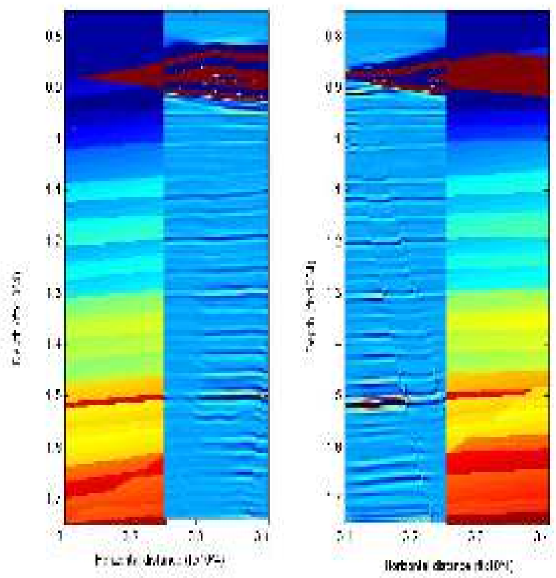

Fig.4. Subsalt imaging.

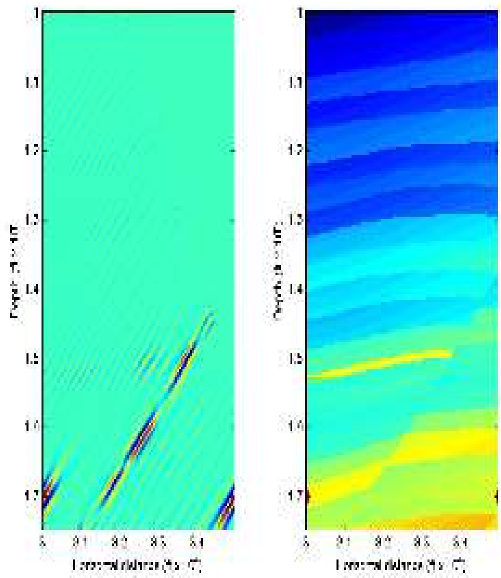

Fig.5. Fault imaging: recovered (left) and real (right)

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